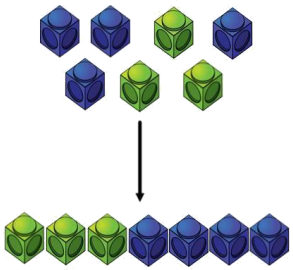
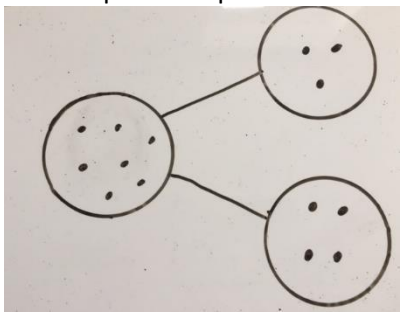
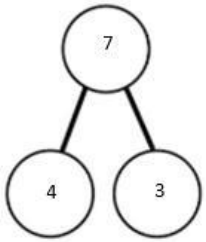
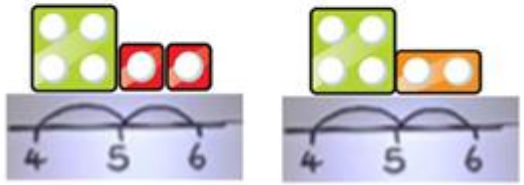
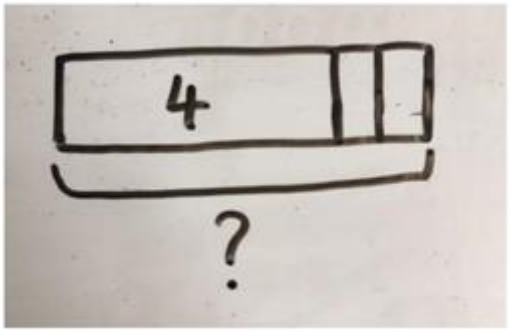
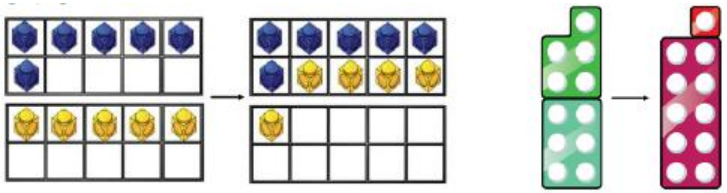
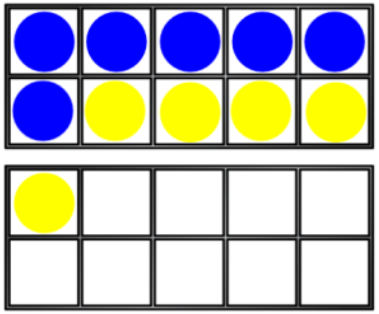
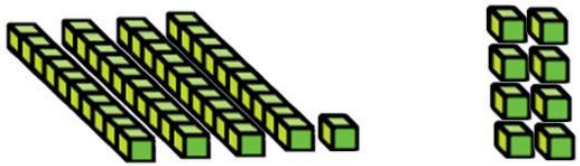
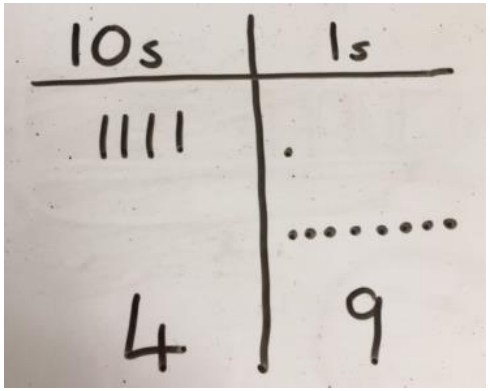
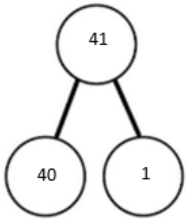
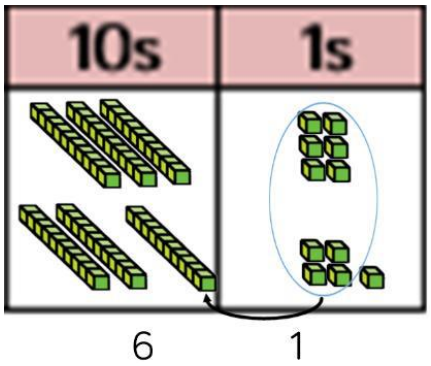
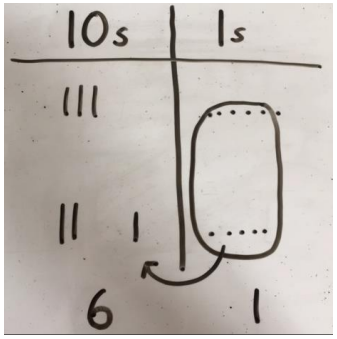
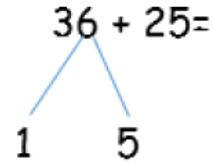
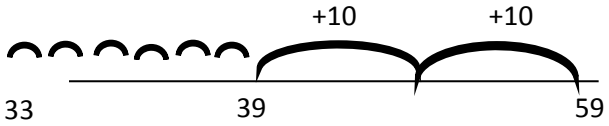
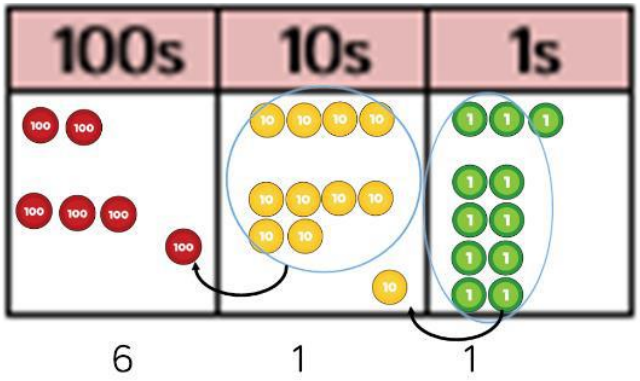
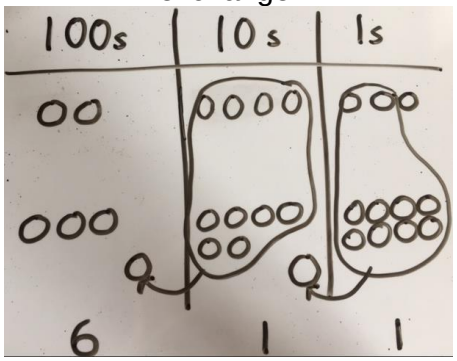
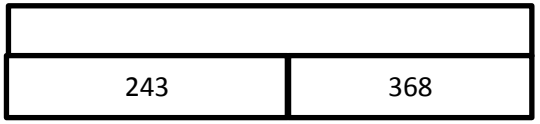
Progression in calculations at Mickle Trafford Village School 2018

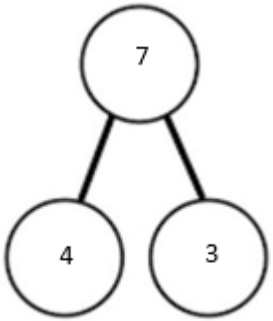
Addition

Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too. E.g. eggs, shells, teddy bears, cars). 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. 	4 + 3 = 7 Four is a part, 3 is a part and the whole is seven.  Children will need to know all 'number stories' about number to 10. E.g. 1+6 = 7 2+5 = 7 3+4 = 7 Etc.
Counting on using number lines using cubes or Numicon.  	A Bar model which encourages the children to count on, rather than count all. 	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 27 

Regrouping to make 10; using ten frames and counters/cubes or using Numicon. 6 + 5 	Children to draw the ten frame and counters/cubes. 	Children to develop an understanding of equality e.g. 6 + □ = 11 6 + 5 = 5 + □ 6 + 5 = □ + 4
TO + O using base 10. Continue to develop understanding of partitioning and place value. 41 + 8 	Children to represent the base 10 e.g. lines for tens and dot/crosses for ones. 	41 + 8  Or written: 20+30 = 50 6 + 3 = 9 26 + 33 = 59
TO + TO using base 10. Continue to develop understanding of partitioning and place value. Using Diene's or Counters 36 + 25 	Children to represent the base 10 in a place value chart. 	Looking for ways to make 10.  30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 36 Formal method: +25 61 1

	Or using a number line to add ones first, then tens. 26+33 = 59  	Or using abstract mental methods.  26 + 33 = 59
Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred. Using Diene's or Counters.  	Chidren to represent the counters or Diene's in a place value chart, circling when they make an exchange.  If the children are completing a word problem, draw a bar model to represent what it's asking them to do.  	Children to use the formal method of addition. 243 +368 611 11 Children to also use formal methods to solve questions involving decimals. 4.3 +36.8 41.1 11 Children should be shown to be lining up their digits in the correct columns. Children can show place value with zeros if they would like to.

Fluency variation, different ways to ask children to solve 21+34:



?	
21	34

Word problems:
In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

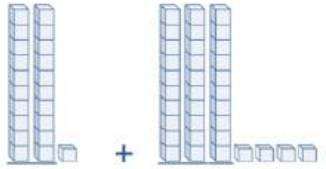
21 + 34 = 55. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

21 + 34 = ☐

☐ = 21 + 34

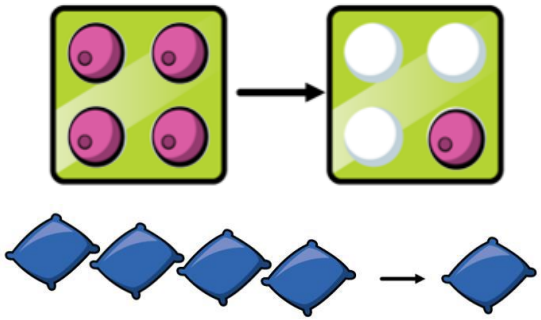
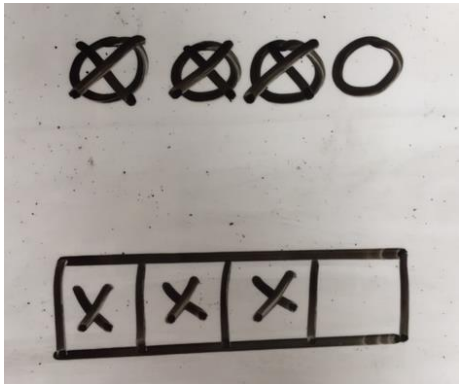
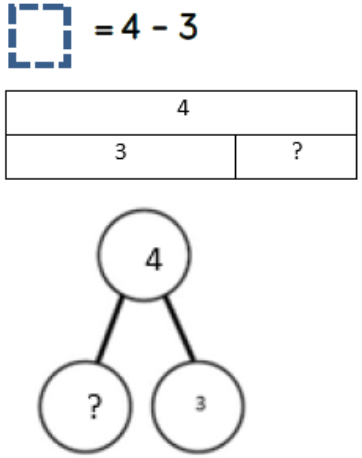
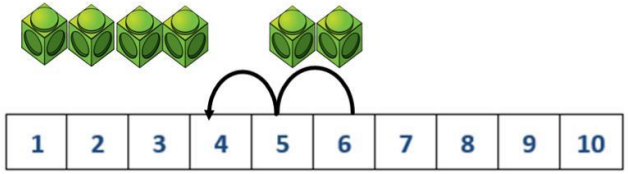
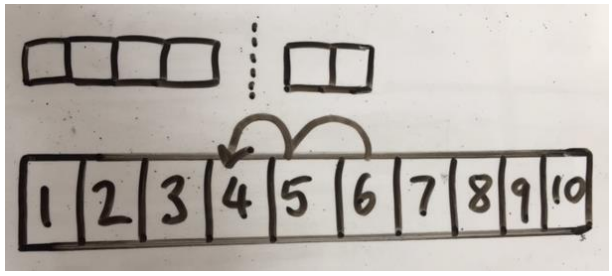
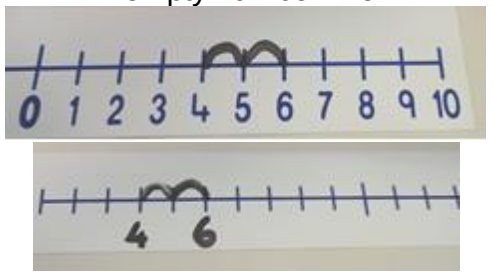
Calculate the sum of twenty-one and thirty-four.

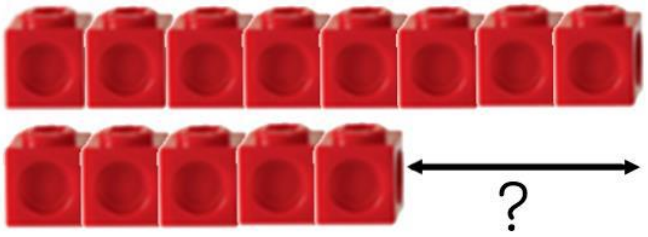
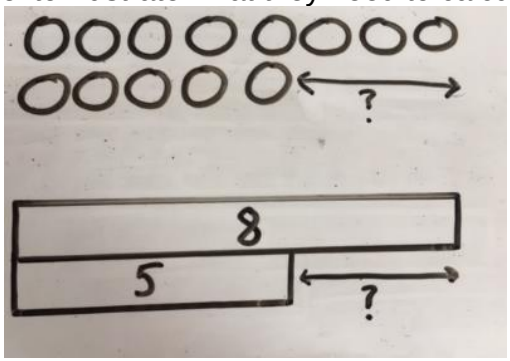
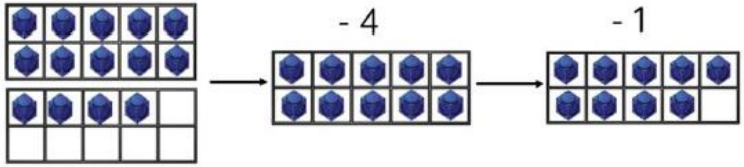
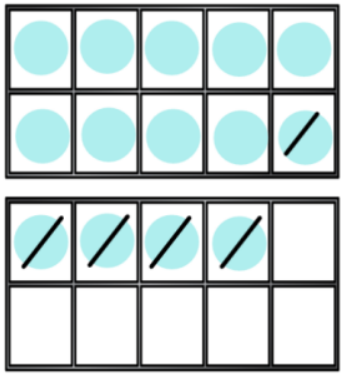
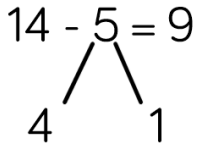


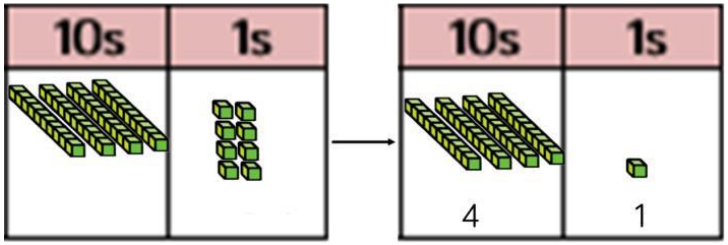
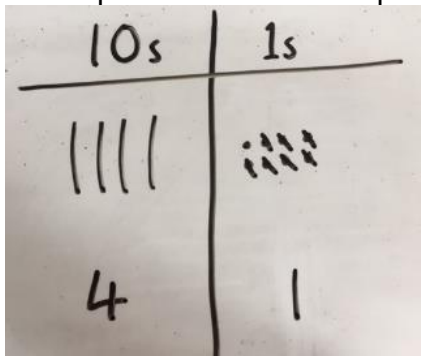
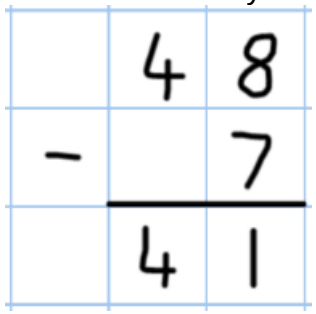
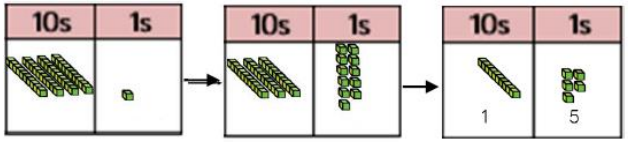
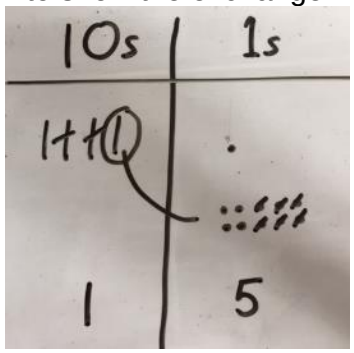
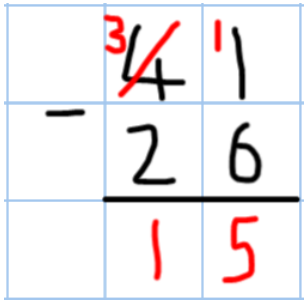
Missing digit problems:

10s	1s
10 10	1
10 10 10	?
?	5

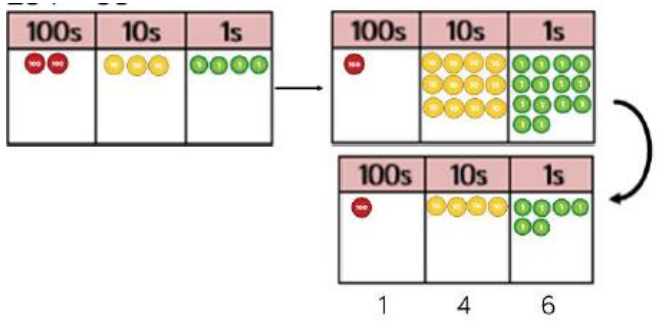
Subtraction

Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). 4 – 3 = 1 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	4- 3 = 
Counting back (using number lines or number tracks) children start with 6 and count back 2. 6 – 2 = 4 	Children to represent what they see pictorially e.g. 	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line 

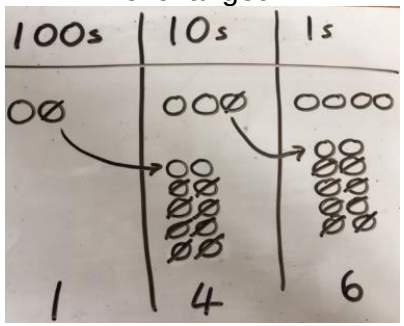
Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used). Calculate the difference between 8 and 5. 	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate. 	Find the difference between 8 and 5. 8 – 5, the difference is ☐ Children to explore why 9 - 6 = 8 – 5 = 7 – 4 have the same difference.
Making numbers within 10 or 20 using ten frames. 14 – 5 	Children to present the ten frame pictorially and discuss what they did to make 10. 	14 – 5 = 9 You also want children to see related facts. E.g. 15-9 = 4 Children to represent how they have solved it. E.g.  5 is made up of 4 and 1, so I can subtract 4 to make 10, and then 1 to get 9.

Column method using base 10. TO - O 48-7  	Children to represent the base 10 pictorially.  	Column method or children could count back 7 mentally. 
Column method using base 10 and having to exchange. TO - TO 41 – 26  1) Start by partitioning 41. 2) Exchange one ten for ten more ones. 3) Subtract the ones, then the tens.	Represent the base 10 pictorially, remembering to show the exchange.  	Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11. 

Column method using place value counters or Diene's.
HTO – TO.
234 – 88



Represent the place value counters pictorially; remembering to show what has been exchanged.

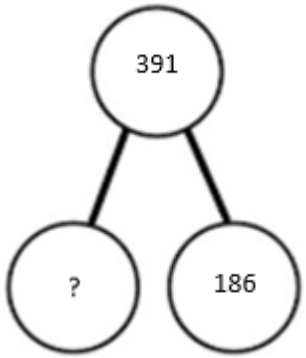


Formal column method. Children must understand what has happened when they have crossed out digits. This should be applied to larger numbers once secure.

$$\begin{array}{r} {}^{2}\not{2}\,{}^{1}\not{3}4 \\ -\ 88 \\ \hline 6 \end{array}$$

Fluency variation, different ways to ask children to solve 391 – 186:

391	
186	?



Raj spent £391, Timmy spent £186.
How much more did Raj spend?

Calculate the difference between 391 and 186.

$\square = 391 - 186$

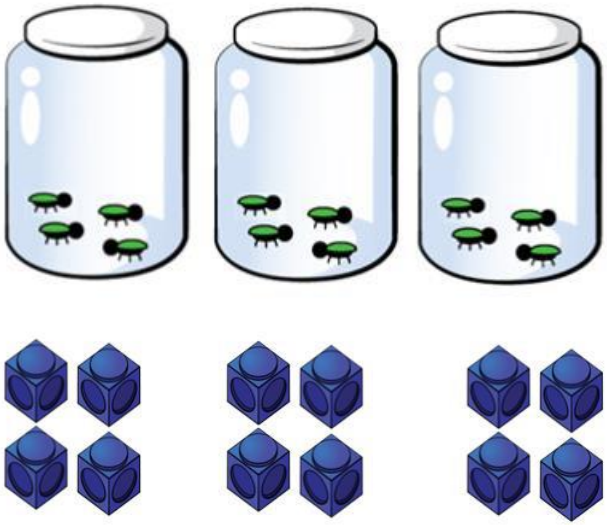
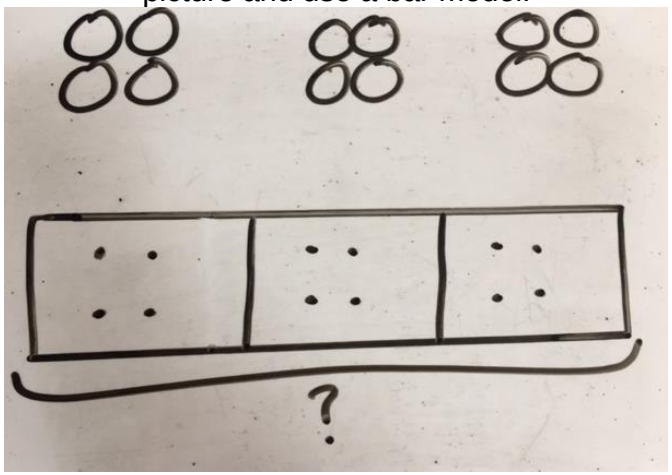
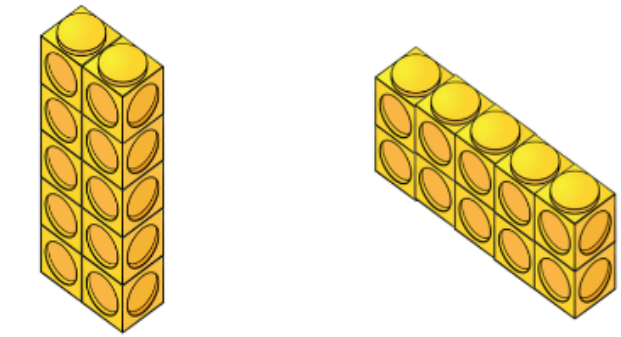
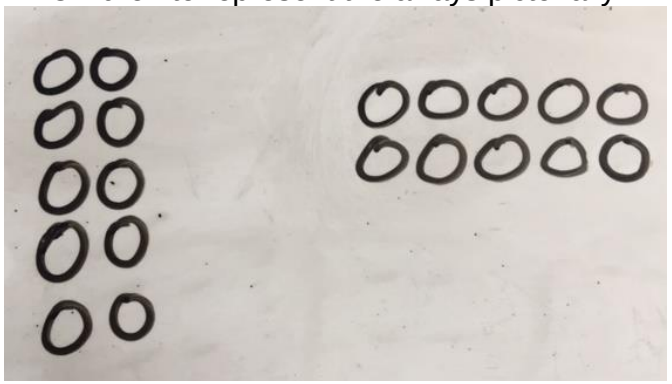
$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

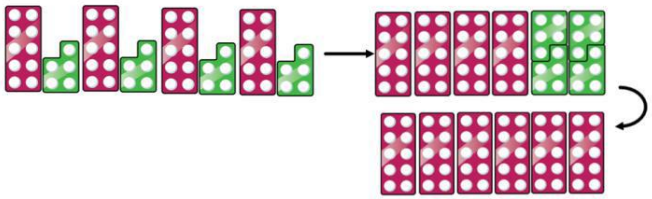
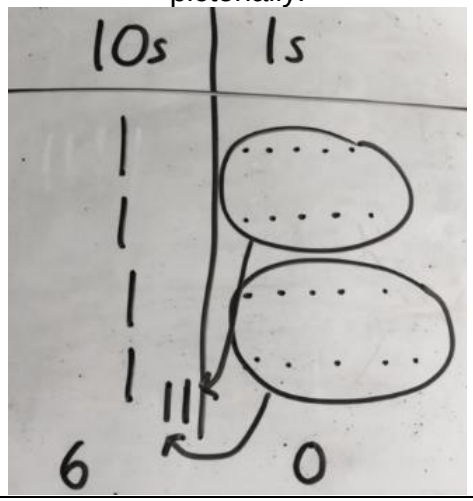
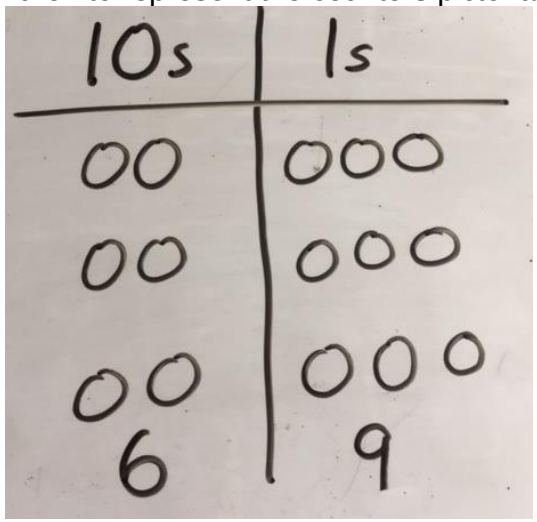
What is 186 less than 391?

Missing digit calculations:

$$\begin{array}{r} 3\ 9\ \square \\ -\ \square\ \square\ 6 \\ \hline \square\ 0\ 5 \end{array}$$

Multiplication

Concrete	Pictorial	Abstract
Repeated grouping/repeated addition 3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group. 	Children to represent the practical resources in a picture and use a bar model. 	3 × 4 = 12 4 + 4 + 4 = 12
Use arrays to illustrate commutativity counters and other objects can also be used. 2 × 5 = 5 × 2 	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. 2 x 5 = 10 5 × 2 = 10 2 + 2 + 2 + 2 + 2 = 10 5 + 5 = 10

Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4 × 15 	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. 4 × 15 10 5 10 × 4 = 40 5 × 4 = 20 40 + 20 = 60
Formal column method with place value counters (base 10 can also be used.) 3 × 23 	Children to represent the counters pictorially. 	Children to record what it is they are doing to show understanding. 3 × 23 20 3 3 × 20 = 60 3 × 3 = 9 60 + 9 = 69 23 × 3 69

Children should partition the question in order to solve it.

6 x 23

6 x 20 = 120
6 x 3 = 18
6 x 23 = 138

Formal written method

$$6 \times 23 =$$

$$\begin{array}{r} 23 \\ \times\ 6 \\ \hline 138 \\ \hline 1\ 1 \end{array}$$

When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract:

To get 744 children have solved 6 × 124.

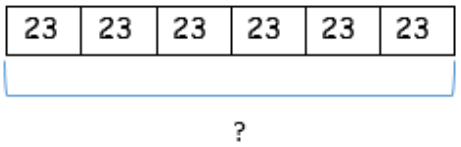
To get 2480 they have solved 20 × 124.

$$\begin{array}{r} 1\ 2\ 4 \\ \times\ \ 2\ 6 \\ \hline 7\ 4\ 4 \\ {\scriptstyle 1\ \ 2} \\ 2\ 4\ 8\ 0 \\ \hline 3\ 2\ 2\ 4 \\ \hline 1\ \ 1 \end{array}$$

Answer: 3224

Fluency variation, different ways to ask children to solve 6 x 23:

Why is 6 x 23 = 23 x 6?



Mai had to swim 23 lengths, 6 times a week.

How many lengths did she swim in one week?

With the counters, prove that 6 x 23 = 138

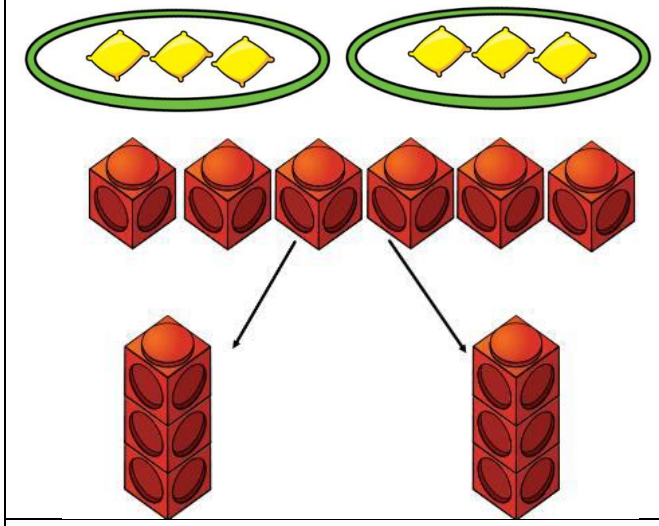
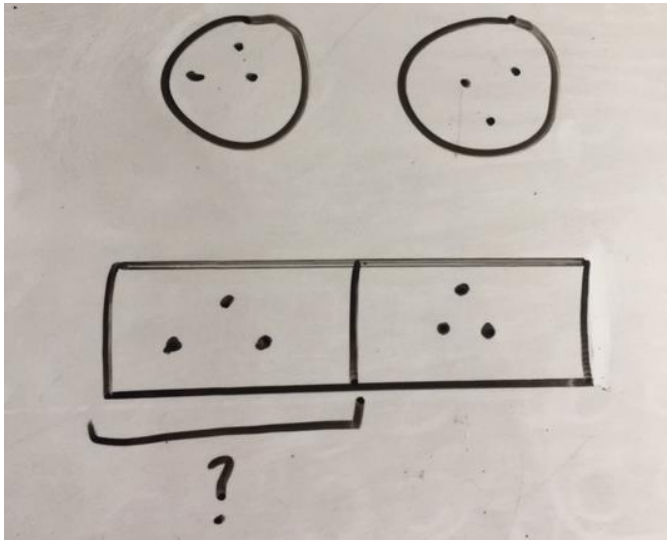
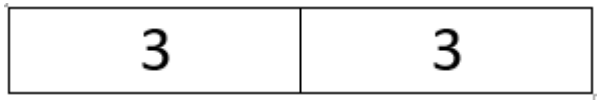
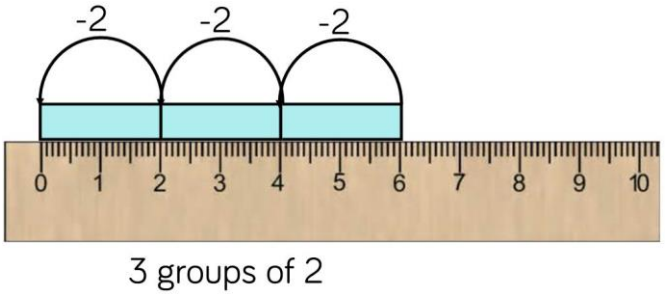
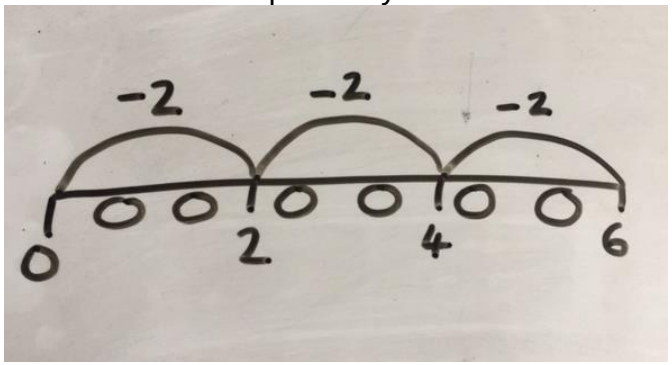
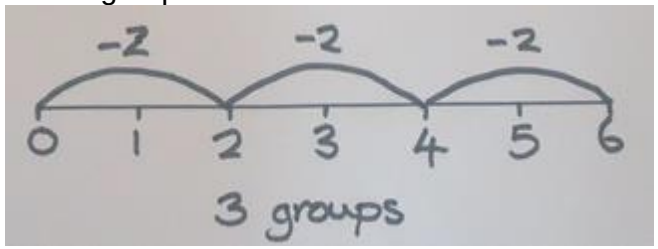
Find the product of 6 and 23

$6 \times 23 =$

$\square = 6 \times 23$

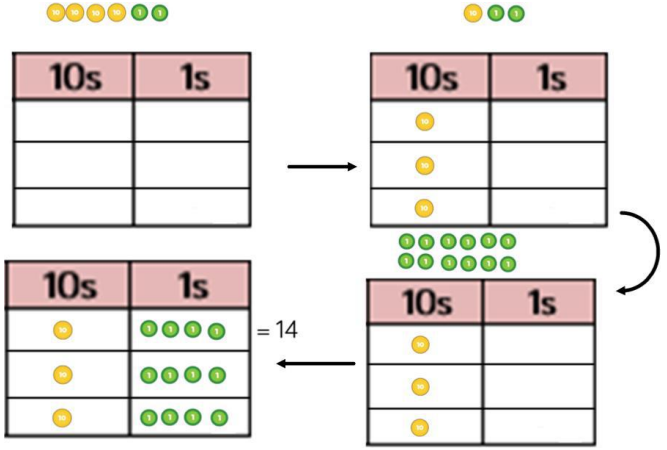
$$\begin{array}{r} 6 \\ \times\ 23 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 23 \\ \times\ 6 \\ \hline \\ \hline \end{array}$$

Division

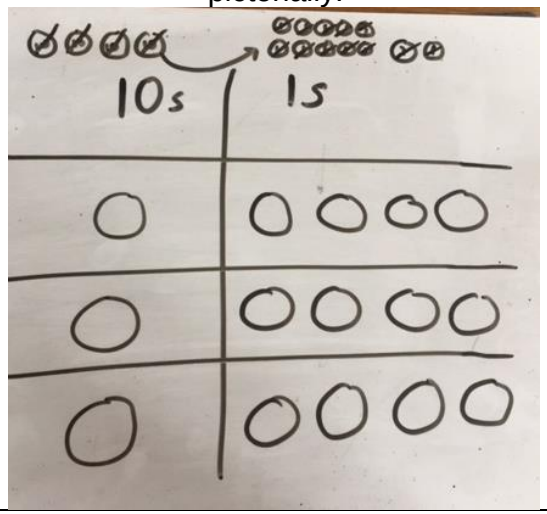
Concrete	Pictorial	Abstract
Sharing using a range of objects. 6 ÷ 2 	Represent the sharing pictorially. 	6 ÷ 2 = 3  Children should also be encouraged to use their 2 times tables facts to share into 2 groups.
Repeated subtraction using Cuisenaire rods above a ruler. 6 ÷ 2 	Children to represent repeated subtraction pictorially. 	Abstract number line to represent the equal groups that have been subtracted. 

Sharing using place value counters.

42 ÷ 3 = 14



Children to represent the place value counters pictorially.



Children to be able to make sense of the place value counters and write calculations to show the process.

42 ÷ 3

42 = 30 + 12

30 ÷ 3 = 10

12 ÷ 3 = 4

10 + 4 = 14

Use of the ‘bus stop method’ to show short division.

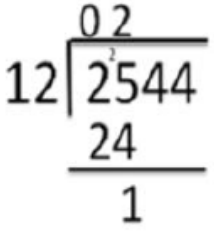
$$5\overline{)615}\quad 123$$

Additionally, children should be shown how remainders can be turned into decimals at the end.

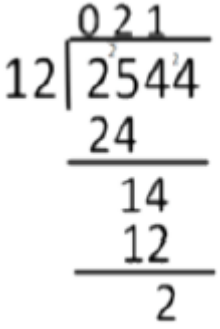
$$4\overline{)542.0}\quad 135.5$$

Long Division

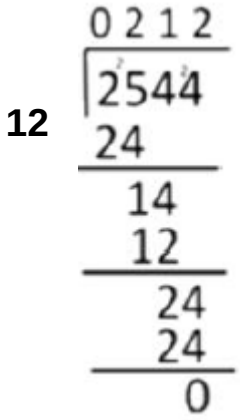
1) exchange two thousands for 20 hundreds, so we now have 25 hundreds.



2) How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left.

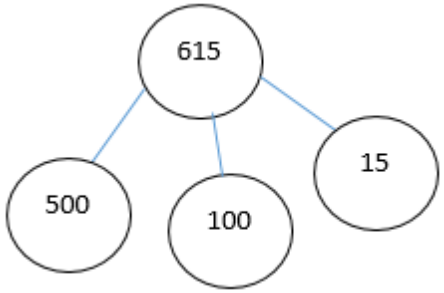


3) Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left.



4) Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.

Fluency variation, different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division?  	I have £615 and share it equally between 5 bank accounts. How much will be in each account? 615 pupils need to be put into 5 groups. How many will be in each group?	$5 \overline{)615}$ $615 \div 5 =$ $\square = 615 \div 5$